Perception of Circular Heading From Optical Flow

William H. Warren, Jr.
Departments of Psychology and Cognitive and Linguistic Sciences, Brown University

Daniel R. Mestre
Centre National de la Recherche Scientifique
Cognition et Mouvement, University d’Aix-Marseille II

Arshavir W. Blackwell and Michael W. Morris
Brown University

Observers viewed random-dot optical flow displays that simulated self-motion on a circular path and judged whether they would pass to the right or left of a target at 16 m. Two dots in two frames are theoretically sufficient to specify circular heading if the orientation of the rotation axis is known. Heading accuracies were better than 1.5° with a ground surface, wall surface, and 3D cloud of dots, and were constant over densities down to 2 dots, consistent with the theory. However, there was an inverse relation between the radius of the observer’s path and constant heading error, such that at small radii observers reported heading 3° to the outside of the actual path with the ground and to the inside with the wall and cloud. This may be an artifact of a small display screen.

How do locomoting observers see where they are going? Gibson (1950: Gibson, Olum, & Rosenblatt, 1955) first showed that translation of an observer through a stationary environment generates a radial pattern of optical flow at the moving eye (Figure 1a), in which the focus of outflow specifies the observer’s direction of self-motion or heading. However, natural locomotion can be more complex than linear translation and often includes curvilinear components. This article examines the perception of the future course of movement on circular paths, which we call circular heading, from patterns of optical flow.

Circular Movement and Optical Flow

We use the term optical flow in Gibson’s original sense to refer to temporal change in the structure of the optic array, which is the pattern of light intensities around a point of observation prior to the introduction of an eye, and retinal flow to refer to change in the pattern of light on the retina. Both optical and retinal flow are typically represented as velocity fields such as Figure 1a, in which each vector denotes the optical velocity of an environmental element, but this is only a partial, instantaneous representation of the flow pattern. For convenience, we describe observer movement in a three-dimensional (3D) Cartesian coordinate system centered at the nodal point of the eye, with Z as the depth axis and optical flow patterns on a planar projection surface normal to the Z axis, although other projection surfaces are formally equivalent.

A general curvilinear movement of the observer may be analyzed instantaneously as the sum of a translation and a rotation (Whittaker, 1944). The resulting retinal velocity field can likewise be analyzed as the sum of two orthogonal components: (a) the translational component, a radial flow pattern corresponding to observer translation (T = T_x, T_y, T_z; Figure 1a), and (b) the rotational component, a solenoidal flow pattern corresponding to observer rotation around any axis through the approximate nodal point of the eye (R = R_x, R_y, R_z), equivalent to a rigid rotation of the environment about the observer.

However, in practice, the rotational component may be due to either an observer rotation (such as a pursuit eye movement) or a curved path of observation. Following Charles’s theorem (Whittaker, 1944), movement of an observer on a circular path around some axis can be described instantaneously as the sum of a rotation R around a parallel axis through the eye and a translation T along the tangent to the path (Figure 2a). Because the instantaneous descriptions of circular movement and translation plus rotation are equivalent, identical velocity fields are generated in the two cases (Figure 1b). Thus, the retinal velocity field is inherently ambiguous.¹

The velocity field produced by circular movement can be understood through the equivalent case of a ground plane rotating beneath a stationary observer. Each surface element has a circular trajectory and an instantaneous velocity tangential to it (Figure 2a). Thus, the flow field can be characterized in the ground plane as a one-parameter family of concentric circles with center C and radius as the parameter, where the circles correspond both to the field’s streamlines and to the

¹ Although eye rotation influences the retinal flow pattern, it does not affect the optic array, whereas translation and circular movement affect both.
Figure 1. Optical velocity field generated by observer movement. (Top panel: Observer translation parallel to ground plane. [Target line indicates heading]. Bottom panel: Circular movement parallel to ground plane. [r = 50 e, target line lies on future path. An identical field is generated by forward translation plus rotation about a vertical axis].)

path lines, or trajectories, of individual elements. The singularity in the field, if it is in view, corresponds to the center of rotation C, and the observer's path on the ground is the circle beneath the eye point, with radius r and curvature κ = 1/r. Projected to the image plane, the family of circles forms a family of hyperbolae (Figure 1b).

Despite their equivalent descriptions, circular movement and translation plus rotation have a distinct functional significance for locomotion. To navigate in a cluttered environment, the observer needs information not only about the instantaneous translation and rotation but also about the future curved path in a geographical frame of reference, assuming current forces are maintained. More complex paths

Streamlines are continuous field lines to which the velocity vectors are tangent at every point and capture the instantaneous structure of the velocity field. A path line is the actual trajectory of an element over time, akin to a time exposure photograph of the flow. A streak line connects the current locations of elements that have previously passed through a given point in the flow field; for example, a plume of smoke represents the streak line from the mouth of a smoke stack. These sets of curves are theoretically distinct, although they coincide in the special case of steady flow, a stationary field in which the velocity at any fixed position in the field does not change over time. For our purposes, fixed positions in the flow field are given in retinal coordinates (see Eskinazi, 1962, pp. 95–102).

Under polar projection, each streamline in the projection plane
of observation can be described by higher order curves, but they require modulations of force and for the moment can be considered as piecewise combinations of osculating circles tangential to the curve.

Thus, we consider three basic classes of observer movement (and their combinations) to contribute to the optical flow pattern: translation, rotation, and circular movement. In the spirit of Gibson (1954, 1968), our working hypothesis is that these classes of observer movement are specified by different classes of optical flow, under some richer description than the velocity field. Our previous experiments showed that observers can perceive translational heading from radial flow patterns with an accuracy on the order of 1° of visual angle (Warren, Morris, & Kalish, 1988) and can decompose the flow field produced by combined translation and rotation to determine translational heading with an accuracy of 1.5° (Rieger & Toet, 1985; Warren & Hannon, 1988, 1990). The present research seeks to demonstrate that circular heading is also perceived from optical flow. Given that the velocity field is ambiguous, how might circular heading be determined?

### Information for Circular Heading

In principle, circular heading could be determined from successive translational headings. Most computational models recover the instantaneous translational and rotational components of observer movement from the velocity field (e.g., Aggarwal & Nandhakumar, 1988; Bruss & Horn, 1983; Heeger & Jepson, 1990; Koenderink & van Doorn, 1981; Longuet-Higgins & Prazdny, 1980; Tsai & Huang, 1981; Verri, Girosi, & Torre, 1989; Waxman & Ullman, 1985). The few reports that explicitly consider circular movement also treat the problem as one of determining instantaneous translation, the tangent to the observer's path (Cutting, 1986; Prazdny, 1981). Circular heading could be derived from this information because the curvature $\kappa$ of the observer's path is equal to the derivative of tangent direction $\tau$ with respect to arc length, $\kappa = d\tau/ds$, or equivalently with respect to time, $\kappa = (d\tau/dt)/v$. Thus, successive tangential directions determined from successive velocity fields would specify path curvature if arc length $ds$ or tangential velocity $v$ were precisely known. In general, these can be determined from optical flow up to a scale factor of eyeheight, so $\kappa$ and $r$ are given in units of eyeheight.

Alternatively, circular heading could be perceived directly. Rieger (1983) showed that higher order accelerative components of the flow field distinguish curved paths of observation from translation plus rotation (specifically, an angular deviation of an element's optical acceleration vector from the translational component of its velocity vector indicates that the observer's path is curved). Three frames are required to define element acceleration. Over time, circular movement with constant curvature yields steady flow, a stationary field with constant streamlines in retinal coordinates, whereas translation plus rotation yields unsteady flow with changing streamlines (see Footnote 2). Successive independent velocity

fields are sufficient to specify the streamlines. In addition, circular movement produces a family of concentric path lines around $C$ that coincide with the streamlines (Figure 3a), whereas translation plus rotation produces path lines around different centers (Figure 3b). Thus, the ambiguity is resolved.

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**Figure 2.** Schematic diagram of circular movement. (Panel a: Heading angle, the visual angle between the target and the point at which the observer's path passes the target; the center of rotation of the observer's path can be determined from two velocity vectors. Panel b: Heading error, the visual angle between the observer's actual path and apparent path at the point they pass the target. $O =$ observation point, $C =$ center of rotation of observer's path, $r =$ radius of curvature of observer's path, $C' =$ center of rotation of apparent path, $r' =$ radius of curvature of apparent path).
Figure 3. Optical path lines of individual elements generated by observer movement. (Top panel: Circular movement parallel to ground plane. \(r = 50\); these path lines coincide with the streamlines of Figure 1b.] Bottom panel: Translation plus rotation about a vertical axis. [Angular velocity equivalent to that in top panel.]

by successive independent velocity fields or if the flow field is permitted to evolve over three or more frames.

Assuming that circular movement has been distinguished from rotation, the flow pattern contains several sources of information for circular heading.

1. **Locomotor flow line.** Lee and Lishman (1977) noted that the observer's future path on the ground plane is specified by what they called the locomotor flow line, that streamline or path line that passes directly beneath the observer. This implies that visible elements are required on or near the locomotor flow line to define it locally. Further, the line is not instantaneously defined in the case of approach to a vertical wall, so we might expect poorer heading judgments in that situation.

2. **Reversal boundary.** The curved path is also specified by the boundary between those elements (or streamlines) on the inside of the path that reverse their horizontal direction of motion on the display screen and those on the outside of the path that do not (refer to Figure 1b). Relying on such information would require a fairly dense flow field to localize the boundary. In addition, the reversal boundary does not generally correspond to the observer's path during approach to a wall if movement stops before impact.

3. **Vector normals.** Rather than relying on a local feature
such as the locomotor flow line or reversal boundary, circular heading could be determined from the motions of a few elements anywhere in the field. To demonstrate this, we begin with the special case of movement parallel to a ground plane, the idealized case for terrestrial locomotion. For convenience, results will be demonstrated in the ground plane, which can then be used to label the visual directions of the optic array or transformed to any desired two-dimensional (2D) coordinate system.\footnote{We thank Jan Koenderink for this approach to the problem.} Projected to the image plane, for example, the family of concentric circles forms a family of hyperbolae (Figure 1b).

Assuming initially that movement is known to be (a) circular with constant curvature and (b) parallel to the plane, then the circular heading is specified by the position of two elements in two frames. This can be seen in Figure 2a, in which the normals to two velocity vectors intersect at the center of rotation C. Given C, the radius and curvature of the observer’s path are determined; if the normals are parallel, the observer’s path is translational.\footnote{The same is true for one element in three frames, where C is given by the intersection of the normals to two successive vectors.} In principle, we can eliminate the first assumption by adding a second velocity field or a third frame, because circular movement is specified by steady flow with concentric circular path lines. Adding more elements or more frames increases redundancy in the estimate of C, which counteracts effects of noise.

We can relax the second assumption and generalize the argument to 3D environments as follows. The paths of elements positioned in three dimensions lie on a set of concentric cylinders whose longitudinal axis is the axis of rotation. Assuming only that the orientation of the axis is known, as it is in terrestrial locomotion, circular heading is again given by two elements in two frames. Each velocity vector has one normal lying in a plane perpendicular to the rotation axis. Together these two normals determine the location of the axis and hence the radius of the observer’s path.

In sum, circular heading is in theory specified by two elements in two frames, assuming that the orientation of the rotation axis is known. Additional frames or velocity fields are required to distinguish circular movement from eye rotation. This hypothesis predicts that very sparse flow fields should be sufficient for accurate perception of circular heading and that features such as a locally defined locomotor flow line or reversal boundary would be unnecessary. The present article examines the effect of the number of elements: manipulation of the number of frames is reported in Warren, Blackwell, Kurtz, Hatsopolous, et al. (1990).

The only empirical research on circular movement is that of Cutting (1986). However, he used displays that simulated the effects of eye counterrotation during movement on a circular path, and his subjects judged tangential heading rather than circular heading. In addition, heading judgments were made with respect to the direction of gaze rather than an environmental referent, so the results are relevant only to a theory of heading perception based on multiple fixations (see Warren & Hannon, 1990).

In the experiments reported here, we presented observers with displays of circular movement relative to random-dot surfaces, similar to a view out of the windshield of a turning car, and asked them to judge whether they would pass to the left or to the right of a target in the display. For present purposes, we manipulated optical flow under free-fixation conditions and assumed that the observer could resolve the effects of any eye rotation; the decomposition of circular movement and eye rotation will be examined directly in future research. The present experiments address three main questions. First, how accurately can observers perceive circular heading? Second, what information in the flow pattern is used to do so? Third, does the ability generalize to different environments? In the course of these experiments, we discovered a systematic tendency for observers to report heading to either the inside or the outside of the actual curved path, and we tried to determine the cause of this heading bias.

### Experiment 1: Circular Movement Parallel to the Ground Plane

In the first experiment, we determined how accurately observers can perceive circular heading and whether it is distinguished from tangential heading. Displays simulated the optical flow pattern that would occur with observer movement on a circular path parallel to a random-dot ground plane, and observers judged whether they would pass to the left or to the right of a simulated target 16 m ahead if they continued on their current path (Figure 2a). The radius of curvature of the path (r), measured in units of eyeheight (e) above the ground plane, varied from 50 e, a relatively tight curve, to 200 e, a nearly translational path: assuming a standing eyehight of 1.6 m, the range was 80 m to 320 m. A value of r = 75 e is similar to the curvature of a highway cloverleaf for a standing observer. The displays were constructed so that if observers could not distinguish circular heading from tangential heading, performance would be near chance in the r = 50, 75, and 100 e conditions.

A secondary issue is the question of possible constant error. Circular paths introduce an asymmetry in movement and thus in the flow pattern, which raises the possibility of a systematic bias in heading judgments. For a variety of reasons, observers might report seeing themselves as heading to the “outside” of the actual circular path (i.e., on a wider curve with a larger radius), which we will call an outside bias. Conversely, they might report heading to the “inside” of the actual curve, which we will call an inside bias. The results are therefore also analyzed for heading bias.

#### Method

**Observers.** Twelve undergraduates were paid to participate in the experiment. All had normal or corrected-to-normal vision, and none had seen our optical flow displays before.
Displays. Displays simulating circular movement parallel to a random-dot ground plane were generated in real time on a Raster Technologies Model One/380 graphics terminal hosted by a VAX 780 computer, and presented on a Sony GDM-1901 monitor with a 60 Hz refresh rate and a medium-short P22 phosphor (10% decay times: R = 1 ms, G = 0.004 ms, B = 0.003 ms). Each display consisted of 56 images with a pixel resolution of 1,280 Horizontal (H) X 1,024 Vertical (V), presented at 15 frames per second. The display was viewed binocularly from the projectively correct distance of 45 cm and subtended 40° H X 30° V. The observer was positioned in a chin rest in a black viewing box with the screen visible through a window in the other end. The ground surface had a simulated dot density of 0.31 dots/e² (0.12 dots/m²); approximately 95 dots were visible on the screen at the beginning of a trial and 45 remained at the end. Eight surfaces with different random-dot configurations were generated. Dots were single white pixels with a luminance of 118 cd/m² on a blue background of 90 cd/m², and did not expand with motion. Dot placement was determined by randomly positioning one dot in each cell of an appropriately scaled grid. To avoid a dense clustering of dots at the horizon, the ground plane was truncated at a simulated distance of 28° e (45 m), akin to a distant cliff edge. The observer's simulated tangential speed in eyeheights per second (e/s) was held constant at 2.4 e/s (3.8 m/s); tangential velocity v is defined by v = rw, where w is angular velocity). These parameters were the same as those in one condition of Warren, Morris, and Kalish's (1988) Experiment 2 on translational heading. The radius of the observer's circular path varied randomly among ±50°, 75°, 100°, and 200° (±80°, 120°, 160°, and 320°, respectively), with positive values indicating a right-hand turn (center of rotation to the observer's right) and negative value indicating a left-hand turn. The axis of rotation was always perpendicular to the ground surface.

Circular heading accuracy was assessed in terms of the heading angle, the visual angle between a target on the ground surface and the point at which the observer's path would pass the target, determined from the simulated observation point at the end of the display (Figure 2a). The heading angle lay in the plane of the observer's circular path, parallel to the ground surface. The target was a vertical 1° line segment, like a post on the ground surface, which appeared in the last frame of the display. This eliminated any information provided by target motion or relative motion between the dots and the target, allowing us to determine the efficacy of the flow pattern itself. The target appeared at a simulated distance of 10° e (16 m) from the final observation point, its location on the ground surface visually specified by the intersection of its base and the surface (Gibson, 1950). The point at which the observer's path would pass the target was determined as the point on the path from which a line to the observation point was perpendicular to a line to the target (Figure 2b). The final time-to-contact with this point (the time remaining at the end of the display before the observer would pass the target) was 4.2° s at all radii. Heading angle varied randomly at ±0.5°, 1.0°, 2.0°, and 4.0° degrees, so on half of the trials the observer was heading to the “inside” of the target, and on the other half, to the “outside” of the target. If observers saw themselves as traveling on a tangential path at the end of the display, performance would fall to chance levels with r ≤ 573°, 287°, 143°, and 72° e for each heading angle, respectively. Thus, the tangent hypothesis predicts chance performance at all heading angles in the r = 50° condition and at all but the largest heading angle in the r = 75° and r = 100° conditions.

Procedure. Observers were instructed to indicate, by pressing one of two response buttons, whether it looked as though they would pass to the left or to the right of the target if they were to continue on their current path. On each trial, the first frame of the display appeared for 1 s as a warning signal, the dots moved for 3.7 s, and the target line appeared in the last frame. The target and the dots remained visible until the observer made a response, followed by a 2-s intertrial interval during which only the blue background was visible. All observers received 10 practice trials with feedback and then 256 test trials without feedback, both in a 1-hr session. Feedback was provided during practice to help orient the observer to the task; we were careful to eliminate known artificial cues from the displays, and we assumed that observers could not learn to use such cues in only 10 trials.

The data were combined across positive–negative radius and positive–negative heading angle for analysis. To calculate heading thresholds, each observer’s data were fit with an ogive by performing a z transformation on the percentage of correct responses and computing a linear regression. The heading angle at which the regression line reached 75% correct was adopted as the threshold. Occasional observers who did not show a clear threshold in a given condition (i.e., those who were below threshold at all heading angles or performed irregularly) were removed from that condition. The mean number of removed observers for all experiments reported here was 0.9 per condition, with a maximum of 3. As a check that heading thresholds reflected the raw data, statistical analyses were performed both on thresholds and on the percentage of correct responses; only thresholds are reported.

Results and Discussion

Accuracy. Mean thresholds are presented as a function of radius in Figure 4. Observers were quite accurate in judging circular heading, with an overall threshold of 1.4° (SD = 0.54°). The heading threshold dropped slightly from 1.8° at r = 50° to 1.2° at r = 200°, and an ANOVA demonstrated a significant radius effect, F(3, 33) = 3.18, p < .05. However, this level of performance is well above chance at all radii, indicating that observers can distinguish circular from tangential heading. It is close to the threshold of 0.7° for translational heading found by Warren, Morris, and Kalish (1988) under the same density and speed conditions.

Bias. A simple measure of heading bias is the total percentage of “outside” responses, which would be at 50% with
no bias. For \( r = 50, 75, 100, \) and \( 200 \), the results were 49.7\%, 46.9\%, 46.5\%, and 48.3\% outside, respectively, with 47.8\% outside overall, which is not significantly different from 50\%, \( t(1) = 0.85, n.s. \) Thus, there is no evidence of bias.

In sum, the first experiment indicates that observers distinguish circular heading from tangential heading on the basis of the flow pattern and can perceive circular heading quite accurately, at least over this range of radii. Thus, it appears that different classes of self-motion such as translational and circular movement are indeed specified by different classes of optical flow and perceived with sufficient accuracy to control locomotion.

**Experiment 2: Flow Field Density**

What optical information do observers use to perceive circular heading? In the introduction, we identified two local properties of the flow field that specify the observer's future path—the locomotor flow line and the reversal boundary. Both of these sources of information require fairly high dot densities to ensure that there are dots on or near the future path, thereby locally defining the features. The vector normal hypothesis, in contrast, proposes that circular heading is specified by the motions of only two elements.

In previous studies of translational movement, we have found that heading thresholds remain accurate with very sparse flow patterns; performance drops significantly only with two dots (Warren, Blackwell, & Kurtz, 1990; Warren et al., 1988). These data follow an \( N^{-0.5} \) rule (\( r > 0.97, \) slope = 4.36, intercept = \(-0.07^\circ\)), where \( N \) is the number of dots, indicating that the visual system makes use of redundancy in the flow pattern provided by added elements, but asymptotes with very few elements. This suggests that observers do not normally depend on local features of the flow field, such as a stationary element at the focus of outflow, but rather on the radial structure of the flow pattern defined over several elements.

Similarly, we predict that circular heading judgments should remain accurate at low dot densities. As just shown, if movement is known to be parallel to the ground plane, circular heading is specified by two elements in two frames. The present experiment manipulated the number of elements in order to compare these three hypotheses.

**Method**

Twelve undergraduate observers, none of whom had seen our displays before, were paid to participate. Displays and procedure were the same as in Experiment 1, with two exceptions. First, to include a sharper curve, the radius was varied among \( \pm 25, 50, 75, \) and \( 100 \) \( m \) (40, 80, 120, and 160 \( m \), respectively). Second, four dot density conditions were included, simulating densities of 0.01, 0.05, 0.13, and 0.31 dots/\( e^2 \) (0.004, 0.02, 0.05, and 0.12 dots/m\(^2 \), respectively), comparable to those of Warren, Morris, and Kalish (1988). For these density conditions, respectively, there were 2 dots visible at the beginning of a trial and 2 visible at the end, approximately 10 at the beginning and 4 at the end, approximately 26 at the beginning and 11 at the end, and approximately 62 at the beginning and 22 at the end. Eight surfaces with different random-dot configurations were generated in each density condition. Tangential speed was constant at 2.4 \( e/s \) (3.8 m/s). There was a total of 256 trials in a 1-hr session.

**Results and Discussion**

**Accuracy.** Performance remained nearly constant over variation in density, although it deteriorated sharply at the smallest radius. Heading thresholds were analyzed in two separate ways because of the small number of trials in each condition. First, thresholds were computed for each density condition, pooling over radius. Mean thresholds were 3.3\(^\circ\), 2.7\(^\circ\), 2.9\(^\circ\), and 2.5\(^\circ\) in the 2-, 10-, 26-, and 62-dot conditions, respectively, with no significant density effect, \( F(3,25) = 1.36, n.s \) (these thresholds are somewhat high because of poor performance at the small radius). However, a correlation and linear regression of threshold on \( N^{-0.5} \) yielded \( r = .89, \) slope = 1.17, and intercept = 2.4\(^\circ\), which suggests that the visual system does make some use of redundancy in the flow pattern.

Second, thresholds were recomputed for each radius condition, pooling over density (Figure 4). The mean threshold rose from 2.1\(^\circ\) at \( r = 100 \) to 6.0\(^\circ\) at \( r = 25 \), yielding a significant radius effect, \( F(3,30) = 6.84, p < .001 \). Post-hoc tests showed that the \( r = 25 \) threshold is significantly higher than all of the others (\( p < .05 \) or better), and \( r = 50 \) is higher than \( r = 100 \) (\( p < .05 \)). There was no Density \( \times \) Radius interaction in the analysis of percentage of correct responses.

In sum, accuracy remains high despite decreasing dot density. As few as two dots are sufficient for accurate judgments of circular heading if travel is known to be parallel to a plane, consistent with our analysis. Thus, locally defined features of the flow pattern such as a locomotor flow line or reversal boundary are unnecessary. It also appears that the visual system exploits redundancy in the flow pattern, as would be expected if additional vector normals improved the estimate of the center of rotation. The unexpected decline in accuracy with decreasing radius was pursued in the next experiment.

**Experiment 3: Small Radius Paths**

The results of the previous experiment indicated a drop in accuracy with small radii of curvature. To explore this effect, we tested small radii between 20 and 50 \( m \), varying observer speed. Radii smaller than 20 were not practicable with our apparatus, because the portion of the flow pattern on the inside of the curve path would be mostly offscreen.

**Method**

Thirteen undergraduate observers, none of whom had previously viewed our displays, were paid to participate. Displays and procedure were similar to those used in Experiment 1, with two exceptions: \( r \) varied randomly among 20, 30, 40, and 50 \( m \) (32, 48, 64, and 80 \( m \), respectively) and was crossed with tangential observer speeds of 0.6, 1.2, 1.8, and 2.4 \( e/s \) (1.0, 1.9, 2.9, and 3.8 m/s, respectively), ranging from a slow walk to a fast run. Final times-to-contact were 16.1, 8.5, 5.5, and 4.2 \( s \) for each speed, respectively, and final target distance was again 10 \( e \) (16 \( m \)). A total of 512 trials was presented in two 1-hr sessions.
Results

Accuracy. There was again an inverse relationship between radius and heading threshold (Figure 5a), with mean thresholds of 4.2°, 2.3°, 1.9°, and 1.6° at r = 20, 30, 40, and 50, respectively. An ANOVA revealed a main effect of radius, $F(3,36) = 21.32, p < 0.001$, but not speed, $F(3,36) = 2.78, ns$, although there was an interaction, $F(9,108) = 2.08, p < .05$. Individual comparisons showed that the speed effect was significant only for the smallest radius, $F(3,36) = 2.87, p < .05$.

Bias. The percentage of outside responses also rose dramatically at small radii, with 72.7%, 60.8%, 52.2%, and 45.9% outside at r = 20, 30, 40, and 50, respectively. To determine whether this outside bias accounts for the drop in accuracy, we examined the percentage of all trials in which inside errors (erroneous “inside” responses) and outside errors (erroneous “outside” responses) occurred. First, the overall percentage of errors increased 11.8%, from 25.3% at r = 50 to 37.1% at r = 20. At the same time, the percentage of outside errors rose 19%, from 10.8% to 29.8%, whereas the percentage of inside errors actually fell 7.2%, from 14.5% to 7.3%. Thus, the rise in thresholds at small radii is completely accounted for by an increase in outside bias.

A more meaningful measure of bias can be obtained in terms of the constant heading error, the visual angle between the observer's actual path and the apparent path at the point it passes the target, determined from the observation point at the end of the display (Figure 2b). To determine heading error, we replotted the percentage of outside responses as a function of uncollapsed heading angle (separating inside and outside trials) and fit the data with an ogive. The point at which this curve crosses the 50%-outside line corresponds to the heading angle at which equal numbers of outside and inside responses were made, and thus provides an estimate of constant heading error. With zero bias, the intersection would fall at a heading angle of 0°. If it falls at an angle of $-2°$, this indicates a 2° inside error: The observer reports being on a circular path that is, on the average, 2° to the inside of the actual path at the point it passes the target. The results of this analysis appear in Figure 5b, which plots constant heading error as a function of radius and speed, with positive values indicating outside errors and negative values indicating inside errors. Mean errors were 3.0°, 1.2°, 0.1°, and $-0.4°$ at r = 20, 30, 40, and 50 e, respectively. An ANOVA reveals main effects of radius, $F(3,36) = 22.58, p < .001$, and speed, $F(3, 36) = 4.01, p < .05$, but no interaction, $F(9, t08) = 0.84, ns$. By t tests, the heading error is significantly different from zero for all speeds at r = 20 ($p < .05$ or better), and for speeds of 0.6 and 1.2 e/s at r = 30 ($p < .01$). Thus, outside error significantly decreases with speed.

Discussion

The results indicate an inverse relationship between radius and outside error, which completely accounts for the rise in thresholds at small radii. However, in practical terms this level of error is quite small. Unlike translational heading, threshold and error values for circular heading depend on target distance, and the observed heading error would decrease substantially as the observer approached the target. Assuming that observers see themselves as traveling on circular paths, the outside error of 3° at 16 m for r = 20 corresponds to an apparent path with a radius of r = 25.16, which would yield an outside error of only 0.7° at 8 m. This level of error should not impair the control of locomotion, although it is important to understand its source.

What could account for the systematic tendency for observers to report seeing themselves as heading to the outside of the actual curve? We conceived of a number of hypotheses, which we attempted to test in subsequent experiments.

1. Center-screen bias. In experiments on translational movement using pointing tasks, a number of researchers have noted a tendency for observers to report heading toward the center of the display screen (Gibson, 1947; Johnston, White, & Cumming, 1973; Llewellyn, 1971). This could account for the outside bias found here because the center of the screen is always to the outside of the observer’s path, and it would...
explain the radius effect because the path moves farther from the center of the screen at smaller radii. To test whether observers exhibit a center-screen bias with our discrimination task, we reanalyzed earlier data on translational heading (from Warren, Blackwell, & Morris, 1989). Over all density and speed conditions, the mean percentage of center-screen responses was 54.9%, not significantly different from chance, t(11) = 1.37, ns; in the comparable high-density condition, the mean was only 52.4%. Thus, it is doubtful that this hypothesis can account for the bias observed with circular movement. The theory also predicts an outside bias regardless of environmental structure, which we tested in Experiments 5 and 6.

2. Target ambiguity. It is possible that with a static post-motion target the location of the target on the ground surface is not clearly specified. If the target appears to be closer to the observer than it actually is, this would explain the outside bias; conversely, if it appears to be farther away, this would produce an inside bias. We tested this hypothesis in Experiment 4 by using a moving target, which provides more information about target position.

3. Visual-vestibular conflict. It is possible that outside bias is the result of a conflict between visual and vestibular information for self-motion. The optical flow pattern specifies that the observer is on a circular path, accelerating toward the center of rotation, whereas vestibular information specifies that the observer is undergoing no acceleration and thus must be on a straight path. Some "compromise" between the two sources of information could yield an outside bias that would increase with smaller radii. This hypothesis predicts an outside bias independent of environmental structure, which was tested in Experiments 5 and 6.

4. Inertial compensation. In actual circular locomotion, the body mass tends to travel along the tangent to the observer's path as a consequence of inertia, and force must be actively applied to maintain a circular path. It is conceivable that a perceptual outside bias could be a means of visually compensating for this inertial effect in locomotor control. However, the inertial effect would increase at higher speeds and thus so should the outside bias, but the results of the present experiment show the opposite result. The bias should also be independent of environmental structure, a hypothesis that was tested in Experiments 5 and 6.

5. Underestimation of flow curvature. It is possible that the visual system is inaccurate in detecting the curvature of optical flow patterns. A systematic underestimation of flow field curvature would explain the outside bias and radius effect, and might account for the reduction of bias at higher speeds, where optical velocities are more easily detected. Again, this would predict an outside bias regardless of environmental structure, which we examined in Experiments 5 and 6.

6. Flow structure. There may be some property of the optical flow pattern itself that contributes to an outside bias, and some possibilities will be discussed later.

7. Small display. Finally, the 40° H display screen necessarily masks more of the flow pattern on the inside than on the outside of the observer's path, increasingly so with smaller radii. This yields a smaller area of flow and fewer elements on the inside of the curve, which might contribute to an outside bias.

The remaining experiments have two goals: (a) to examine the generality of circular heading judgments under a variety of target and environmental conditions and (b) to test these possible explanations of heading bias.

Experiment 4: Target Effects

Under natural conditions, observers generally navigate with respect to continually visible objects rather than a postmotion target. To examine the effects of using a visible target, we repeated the basic experiment for two other conditions: (a) a moving target on the ground surface and (b) a moving target with no surrounding flow.

A moving target may provide additional information about circular heading that is unavailable in the flow of surface elements. First, there is motion parallax between the target and the foreground dots, called target shear. If observers depend on this information, performance should improve with the moving target but should drop with the target alone. Warren, Morris, and Kalish (1988) found no such improvement for translational heading with the addition of target shear. Second, it is a simple matter to extrapolate the trajectory of the target itself to determine whether it will pass to the right or to the left of the observer. Third, observers could rely on a reversal in the horizontal direction of target motion, a manifestation of the reversal boundary. A lateral reversal at some point in the target's trajectory specifies that the observer will pass to the outside of the target, whereas the absence of a reversal specifies a path to the inside. If observers normally depend on these last two sources of information, there should be no difference between the Moving Target and the Target Only conditions, but both should be better than the Postmotion Target condition of Experiments 1 and 3.

This experiment also allowed us to test the target-ambiguity hypothesis as an explanation of outside bias. A moving target should remove any ambiguity about the position of the target on the ground surface, for the target and elements near its base move together with the same velocity. Thus, if target ambiguity is the source of outside bias, it should be reduced in the Moving Target condition.

Method

Observers. Two groups of observers were paid to participate, 10 in the Moving Target condition and 9 in the Target Only condition. None had previously seen our displays.

Displays. Displays were similar to those of Experiments 1 and 3. Dot density was 0.31 dots/e² (0.12 dots/m²), tangential speed was 1.2 e/s (1.9 m/s), and radii varied among 20, 50, 75, and 100 e (32, 80, 120, and 160 m, respectively). In the Moving Target condition, the target and the random-dot ground surface appeared in the first frame of the display, and the target moved as though it were an object on the surface, including vertical (but not horizontal) expansion. It stopped in the last frame of the display in the same final position and size as in the previous experiments. In the Target Only condition, the motion of the target was the same, but the random-dot surface was deleted.
Procedure. The procedure was the same as before, except that observers in the Target Only condition were given two sets of 10 practice trials with feedback. In the first set, they viewed a moving target on the ground surface in order to demonstrate the situation of self-motion over the ground. They were then told that from then on they would only be able to see the target, like a luminous pole in the dark, but that the task remained the same; they then received a second set of target-only practice trials. A total of 256 test trials were presented in each condition, without feedback.

Results and Discussion

Accuracy. Heading thresholds for the Moving Target and Target Only conditions are presented in Figure 6a, together with those for the Postmotion Target condition from Experiment 3 (r = 20, 50 e, v = 1.2 e/s) and Experiment 1 (r = 75,100 e, v = 2.4 e/s; although observer speed was greater in that experiment, there were no speed effects for r ≥ 30). There was no statistical difference between the results from the Moving Target condition and the Postmotion Target condition at small radii, F(1,21) = 2.69, ns (despite the apparent difference for r = 20 in Figure 6a), nor at large radii, F(1,20) = 4.01, ns. However, performance was significantly better in the Target Only condition than in the Moving Target condition, F (1,17) = 6.78, p < .05. Separate ANOVAs indicated a radius effect in both the Moving Target condition, F(3,27) = 35.48, p < .001, and the Target Only condition, F(3,24) = 22.07, p < .01. Observers were extremely accurate with the target alone, yielding a mean threshold of 0.5° for the larger three radii but 2.1° at r = 20.

Bias. There were significant outside errors of 3.0° with the moving target (p < .05) and 2.9° with the postmotion target (p < .001) at r = 20, but a nonsignificant error of −0.9° with the target alone (Figure 6b). Separate ANOVAs showed no difference between the Moving Target and Postmotion Target conditions (r = 20,50), F(1, 21) = 0.51, ns, but a significant difference between the Moving Target and Target Only conditions, F(1,17) = 6.78, p < .05.

This pattern of results indicates that observers rely on the same information in the Postmotion Target and Moving Target conditions—the optical flow of surface elements—but adopt a different strategy in the Target Only condition. The addition of a moving target on the ground surface did not reduce the level of outside bias. Thus, observers did not take advantage of target shear, target trajectory, or target reversal, even though they were available, but instead depended on the surrounding optical flow pattern. However, in the Target Only condition they were able to use target motion and improved their performance. It thus appears that the visual system depends on the optical flow regardless of the presence of a moving target. In addition, this indicates that outside bias cannot be due to ambiguity in the position of the target, because the same level of bias was found in the Moving Target condition when target location was clearly specified.

Experiment 5: Circular Movement Toward a Wall

Up to this point, we have been examining the special case of movement parallel to a ground plane, and it is important to determine whether the perception of circular heading generalizes to other environments. This experiment tests a circular approach to a vertical wall surface. In the case of observer translation, Warren, Morris, and Kalish (1988) found comparable heading accuracies with a ground surface and a wall.

In theory, if the orientation of the rotation axis is known, circular heading toward a wall can be determined from two elements in two frames. However, the instantaneous velocity field generated by circular approach to a wall is quite different from that for the ground. The curved structure of the flow pattern is replaced by an approximately radial structure emanating from a "pseudofocus" of outflow located to the inside of observer’s actual impact point (Figure 7a). The visual angle between the impact point and this pseudofocus increases with smaller radii, as well as with lower speeds because of a greater final distance to the wall. Note also that there is no instanta-
Figure 7. Optical velocity fields generated by circular movement. (Top panel: Wall surface. \( r = 50 \) e, viewed at start of display with tangential heading perpendicular to wall. Circle indicates impact point.) Bottom panel: 3D cloud. \( r = 50 \) e, viewed at start of display. Circle indicates point on observer's path in middle of cloud.)
neously defined locomotor flow line or reversal boundary; the curvature of the flow pattern only becomes apparent when path lines evolve over time. Thus, the case of approach to a wall tests the generality of circular heading judgments and explores the type of flow structure required. We again used Postmotion Target and Moving Target conditions to test the effects of a moving target.

Method

There were 11 undergraduate observers in the Postmotion Target condition and 8 in the Moving Target condition. All were paid for their participation, and none had seen our displays before. Displays simulated a circular path of movement around a vertical axis toward a vertical random-dot wall. The tangent to the path at the observation point was perpendicular to the wall at the beginning of a trial, such that the observer always started out heading straight toward the wall and then veered off. The simulated starting distance from the wall was 13.1 e (21 m). Dot density was 3.0 dots/e2 (1.17 dots/m2) with approximately 200 dots visible at the start of a display, and eight different random-dot surfaces were used. In the Postmotion Target condition, the target appeared on the wall in the last frame of the display. Observers were instructed to judge whether they would hit the wall to the left or to the right of the target. Radius varied among 25, 50, 75, and 100 e (40, 80, 120, and 160 m, respectively), and tangential speeds were 0.6, 1.2, 1.8, 2.4 e/s (1.0, 1.9, 2.9, and 3.8 m/s, respectively). This resulted in simulated final target distances of approximately 11.2, 9.4, 6.9, and 5 e (18, 15, 11, and 8 m, respectively), and final times-to-contact of approximately 19, 8, 4, and 2 s. Radius and speed varied randomly, for a total of 512 trials in two 1-hr sessions. Otherwise, the procedure was the same as before.

An abbreviated Moving Target condition was also used to check the effects of a visible moving target. In this case, the target appeared in the first frame of the display and moved as though it were on the wall; its final size and position were the same as in the previous condition. Radii were 25, 50, and 100 e (40, 80, and 160 m, respectively) and speed was 1.2 e/s (1.9 m/s), with a total of 288 trials in a 1-hr session.

Results

Accuracy. In the Postmotion Target condition, heading judgments for the wall were quite accurate but again deteriorated at small radii (Figure 8a), with mean thresholds of 4.2°, 2.4°, 1.5°, and 1.3° at r = 25, 50, 75, and 100 e, respectively. In contrast to the ground surface, however, accuracy tended to decrease at higher speeds (and thus at shorter times-to-contact and smaller target distances). An ANOVA confirmed main effects of radius, F(3, 30) = 55.40, p < .001, and speed, F(3, 30) = 6.23, p < .01, with no interaction, F(9, 90) = 1.94, ns. Comparing the v = 1.2 e/s condition from the present experiment with that for the ground in Experiment 3 (which had a final target distance of 10 e and a final time-to-contact of 8.5 s), respective mean thresholds for the wall with r = 25 and 50 were 3.4° and 2.2° and for the ground with r = 20 and 50 were 5.0° and 1.5°. Thus, accuracy for the wall was close to that for the ground.

Accuracy improved slightly in the Moving Target condition, F(1,17) = 9.91, p < .01 (v = 1.2 e/s only), with no Condition × Radius interaction, F(2,34) = 0.44, ns. Within the Moving Target condition, there is again a radius effect, F(2, 14) = 11.8, p < .01. To compare this with the ground surface, thresholds for the wall were plotted together with those for the ground in the comparable Moving Target condition from Experiment 4 (Figure 9a). Performance was significantly better with the wall, F(1,17) = 9.91, p < .01, with no interaction.

Bias. Interestingly, the bias with the wall was opposite to that with the ground surface (Figure 8b): Observers exhibited a large inside bias that increased at higher speeds, reporting that they would hit the wall on the inside of the actual impact point. In the Postmotion Target condition, the mean inside errors were −3.1°, −2.5°, −1.4°, and −1.2° at r = 25, 50, 75, and 100 e, respectively. All errors in Figure 8b were significantly inside (p < 0.05 or better) except for the slowest speed, at which there was no bias. An ANOVA on heading error revealed a strong effect of speed, F(3, 30) = 50.36, p < .001, a radius effect, F(3, 30) = 10.43, p < .001, and an interaction, F(9, 90) = 5.77, p < .001. Individual comparisons showed a
radius effect in each speed condition except at the slowest speed. There was clearly a qualitative difference between the significant inside error with the wall and the significant outside error with the ground.

In the Moving Target condition there was also a significant inside error at all radii ($p < .01$), ranging from $3.4^\circ$ at $r = 25$ to $-0.7^\circ$ at $r = 100$ (Figure 8b), and a significant radius effect, $F(2,14) = 9.96$, $p < .01$. This level of bias was no different than that with the Postmotion Target ($\nu = 1.2 \text{ e/s}$), $F(1,17) = 0.69$, ns. Even though the addition of a moving target appears to reduce heading thresholds slightly, it does not reduce the level of bias. Thus, it does not seem that observers are making significant use of target motion information. Heading errors for the wall were compared with those for the ground in the comparable Moving Target condition from Experiment 4 in Figure 9b. There was a highly significant difference between them, $F(1,16) = 19.96$, $p < .001$, confirming the difference between outside and inside errors.

**Discussion**

Perception of circular heading appears to generalize to an approach to a wall surface, with heading accuracies similar to those for movement over a ground surface. This is consistent with our analysis that similar heading information is available in both environments. However, the effects of these surfaces differ in an important way, for although movement over the ground elicits an outside bias that decreases with speed, movement toward the wall elicits an inside bias that increases with speed. This has implications for several of the bias hypotheses.

First, the results contradict the notion of a center-screen bias, because the inside bias observed here occurs toward the edge of the screen. Second, the inside bias contradicts the vestibular hypothesis, which predicts an outside bias in all cases because of the absence of vestibular information for acceleration. Third, it likewise contradicts the inertial hypothesis, which also predicts an outside bias under all conditions. Of course, the results would not present a problem for either of these hypotheses if observers were seeing the displays not as self-motion but as a wall approaching them; this was remedied in Experiment 6 with a 3D cloud of elements, which induced a strong sense of self-motion. Fourth, the inside bias contradicts the hypothesis that observers underestimate the curvature of the flow pattern, because this also predicts an outside bias under all conditions. Thus, the occurrence of the inside bias casts doubt on four of the seven hypotheses.

What can account for this shift to an inside bias? One possibility is the structure of the flow pattern with a circular approach toward a wall. As noted here earlier, the instantaneous velocity field in this case has a radial structure with a pseudofocus of outflow lying to the inside of the actual impact point (Figure 7a). Because the visual angle between the focus and the impact point increases at smaller radii, this could also account for the radius effect. To test this hypothesis, we measured the visual angle between the focus and the impact point at the end of the display and performed a linear regression of heading error on the focus–impact angle. The correlations are generally strong ($r = .78, .19, .95, .99$, for $\nu = 0.6, 1.2, 1.8, 2.4 \text{ e/s}$, respectively), but the slopes of the regression lines are much less than 1 ($b = -0.04, 0.04, 0.26, 0.58$, respectively). Although the effect is in the right direction, the hypothesis predicts a much greater inside bias than is actually observed. In addition, the observed speed effect is opposite to that predicted; whereas the final focus–impact angle increases at lower speeds, the observed inside bias actually decreases. Thus, observers do not simply use the pseudofocus to determine circular heading, although it remains possible that it exerts a small biasing influence on observers’ judgments.

Another possibility is a reliance on element reversal. Normally, the target and other elements will undergo a lateral reversal in motion at some point in their trajectories if the observer’s path passes to the outside of them and will exhibit no reversal if the path passes to the inside. However, for certain combinations of radius and heading angle in this experiment, the display stopped before the target (and some of the inside elements) had reversed. Specifically, on outside trials, reversal occurred only for a target or element $\geq 1^\circ$ to the inside of the impact point with $r = 50$, and $\geq 2^\circ$ with $r = 25$. Thus, if observers were relying on target or element reversal, this would result in an inside bias under these conditions and correct outside responses otherwise. However, the results show a greater inside bias than these reversals can explain: There was a significant inside bias at both large and small radii, the heading error at all radii was larger than predicted, and reanalysis of the Moving Target condition showed an inside bias under conditions with a clear target reversal.

The major difference between a wall and a ground surface is the absence of variation in depth. We have previously shown that the visual system exploits variation in depth to decompose the translational and rotational components of the flow field (Warren & Hannon, 1988, 1990). To examine whether this could account for the inside bias and to test generalization to a more complex environment, we restored depth differences by presenting circular movement through a 3D cloud of elements.

**Experiment 6: Circular Movement Through a 3D Cloud**

The most general case is provided by a 3D volume of elements, which generates a highly discontinuous flow field with relative motion between neighboring elements at different depths (Figure 7b). The instantaneous velocity field again has a roughly radial pattern centered on a pseudofocus to the inside of the actual path. The angular separation between the pseudofocus and the heading point increases with smaller radii and would predict a large inside bias. Alternatively, the visual system might use the relative motion produced by depth differences in a cloud to reduce the bias. In addition, because the cloud produces a strong impression of self-motion, this experiment provides a stronger test of the vestibular and inertial hypotheses.

**Method**

Ten undergraduate observers, none of whom had seen our displays before, were paid for their participation. Elements were randomly...
positioned in a 3D cube whose near and far faces were at simulated distances of 6.25 and 25 e (10 and 40 m) at the beginning of a trial and whose sides were offscreen. Eight different random-dot clouds were generated. Because the location of a postmotion target within the cloud cannot be visually specified, we used only a moving target, whose final position and size were the same as with the ground surface. Radii were 25, 50, 75, and 100 e (40, 80, 120, and 160 m, respectively), observer speed was constant at 1.2 e/s (1.9 m/s), and final time-to-contact was 8.5 s. Each observer received 256 trials in a 1-hr session. Otherwise, the procedure was the same as before.

Results and Discussion

Accuracy. Heading judgments were highly accurate (Figure 9a), with mean thresholds of 1.6° at r = 25 to 0.6° at r = 100, although an ANOVA again showed a radius effect, $F(3,27) = 7.70, p < .01$. Accuracy with the cloud was significantly better than that with the ground, $F(1,18) = 10.66, p < .01$, but no different from that with the wall, $F(1,16) = 0.03$, ns, in the comparable Moving Target conditions ($v = 1.2$ e/s).

Bias. The mean constant heading error ranged from $-1.5°$ at r = 25 to $-0.9°$ at r = 100 (Figure 9b) and was significantly inside at all radii ($p < .05$ or better). Again, there was a radius effect, $F(3,27) = 2.94, p < .05$. The mean error with the cloud was significantly different from that with the ground, $F(1,18) = 16.49, p < .001$, and the wall, $F(1,16) = 4.50, p < .05$, in the comparable Moving Target conditions. However, the only difference from the wall occurred at r = 25 ($p < .05$).

We do not believe that observers were relying on target motion, because the addition of a moving target did not affect the bias that occurred with either the ground or the wall. The presence of depth differences appears to reduce the bias slightly, compared with the wall, but nevertheless a significant bias remains.

To test whether observers were relying on the pseudofocus of outflow, we performed a linear regression of heading error on the visual angle between the heading point and the pseudofocus at the end of a trial. The correlation was high, $r = .94$, but the slope was only 0.07, indicating that the observed inside bias was far less than that predicted by the pseudofocus. This confirms that observers do not use the roughly radial outflow pattern to perceive circular heading, although it is possible that the pseudofocus has some lesser influence on heading judgments.

Thus, perception of circular heading appears to generalize to a complex 3D environment with good accuracy. The small but significant inside bias allows us to reject the vestibular and inertial hypotheses, because there was a strong impression of self-motion. It also allows us to reject the hypothesis that flow field curvature is underestimated by the visual system. All of these hypotheses predict a systematic outside bias, contrary to the present findings. The results also indicate that the inside bias elicited by the wall cannot be explained by the absence of variation in depth, because a significant inside bias remains when the cloud is used.

Experiment 7: Circular Movement With Half a Ground Surface

Two possible explanations for the inverse relation between radius and heading error remain: (a) some unspecified property of the flow pattern or (b) an artifact of a small display screen. The main consequence of using a small display screen is that flowing on the inside of the observer's path is masked by the edge of the screen, producing a smaller area of flow and fewer visible elements. If this is the explanation, then eliminating the flow on the inside of the path should increase heading bias, and eliminating the flow on the outside of the path should decrease the bias. In the last experiment, we tested this hypothesis by deleting the dots from a ground surface on either the inside or the outside of the observer's path. If the hypothesis is correct, eliminating dots from the inside of the path should increase the outside error and vice versa.
Method

Nine undergraduate observers who had not previously seen our displays were paid to participate. Displays of circular movement parallel to a ground plane were similar to those in the previous experiments, with radii of 20, 50, 75, and 100 cm (32, 80, 120, and 160 m, respectively), a tangential speed of 1.2 e/s (1.9 m/s), a dot density of 0.31 dots/e2 (0.12 dots/m2), and a poststimulation target at 10 e (16 m). In the Outside condition, dots were present only on the outside of the observer’s future path and were deleted on the inside of the path; this exaggerated the masking effect of a small screen. Conversely, in the Inside condition, dots were present on the inside of the path but deleted from the outside; this introduced an analogous masking effect for outside dots. To ensure that the dot margin did not provide information about radius, dots appeared only to the outside of the largest radius (100 cm) or to the inside of the smallest radius (20 cm). The procedure was the same as before, with a total of 256 trials.

Results and Discussion

An ANOVA on heading error revealed no difference between the Inside and Outside conditions. Respective mean heading errors for r = 20, 50, 75, and 100 cm were −0.44°, 0.64°, 0.53°, and 0.34° in the Outside condition and 3.01°, 1.8°, −0.87°, and −0.61° in the Inside condition. Individual t tests showed that the only marginally significant bias was the outside error of 3.01° in the Inside condition at r = 20 cm (p < .10), due to large variability. This is essentially the opposite of what would be expected if masking of inside dots were causing the outside bias. Thus, the reduced amount of flow on the inside of the path that occurs with a small display screen does not appear to explain the outside bias observed with a ground surface at small radii.

General Discussion

Let us try to summarize this plethora of results. Most important, observers can distinguish circular from tangential heading under all conditions and can perceive the former with an accuracy sufficient for the control of locomotion (thresholds of 1.5° for r ≥ 50). Perception of circular heading generalizes from a ground surface to a wall surface and a 3D cloud with comparable accuracy. There is a significant heading error at small radii of curvature, but this error is relatively small (3° or less at 16 m), decreases as the observer approaches the target, and thus should not impair locomotor control. Most interesting, heading judgments remain accurate with as few as two elements, consistent with our analysis.

The results allow us to evaluate the proposed explanations of the inverse relation between heading error and radius:

1. The center-screen bias hypothesis can be rejected because there is no evidence of such a bias during observer translation and because the inside bias observed for the wall and the cloud is actually toward the edge of the screen.

2. The target ambiguity hypothesis is inconsistent with the fact that the Moving Target condition did not reduce the bias for either the ground or the wall.

3. The vestibular hypothesis is contrary to the inside bias found using the wall and the cloud. Although this might be attributed to a reduced sense of self-motion with the wall, there was also an inside bias for the cloud despite a clear impression of self-motion.

4. The inertial hypothesis can be rejected for the same reason. In addition, whereas this hypothesis predicts that outside bias for the ground should increase at higher speeds, it actually decreases.

5. Underestimation of flow curvature is also contrary to the inside bias observed for the wall and the cloud.

6. The flow structure hypothesis proposes that the bias can be accounted for by some property of the flow pattern that depends on environmental layout. One candidate was element reversal, but this could not account for the magnitude of the observed errors. Another candidate was the appearance of a pseudofocus of outflow to the inside of the observer’s path with the wall and cloud, which might explain the inside bias. However, simple use of the pseudofocus to indicate heading predicted a much greater constant error than was observed and was inconsistent with the reduction in error observed at lower speeds.

It remains possible that the structure of the flow pattern has a biasing influence on heading judgments. For example, with the wall and the cloud (Figure 7) the larger flow vectors are predominantly horizontal, and the environment appears to slip laterally, which could bias the observer toward the inside of the curve. Conversely, with the ground surface (Figure 1b), the large vectors tend to radiate from a point slightly to the outside of the curve and could bias the observer toward a linear path. Both of these influences are exaggerated by a small display that masks large vectors elsewhere in the field that would provide countervailing information.

7. The fact that a small display masks more of the flow on the inside than on the outside of the observer’s path is by itself an inadequate explanation, given that deleting all dots on the inside or outside of that path had no effect on the bias. However, a small display may yield other artifacts, such as limiting the view ahead along the future path, promoting a tendency to look straight ahead, or masking large vectors as just noted. This might be tested by reducing display size with smaller masks in an attempt to inflate the heading error.

Thus, the present experiments allow us to reject most of the proposed hypotheses, but do not provide a clear explanation of the small but systematic inverse relation between radius and constant error. Any explanation of this effect must account for the fact that it depends on environmental structure and observer speed, yielding outside errors that decrease with speed for the ground and inside errors that increase with speed for the wall and cloud. Thus, heading bias must result in part from differences in flow structure under these conditions. We suspect that the bias is an artifact of the restricted view of the flow pattern presented on a small display screen.

The results do allow us to draw some conclusions about the information used to perceive circular heading. First, it appears that a local feature of the flow pattern such as the locomotor flow line or the reversal boundary is unnecessary. Heading accuracy remains high with sparse flow fields in which such features are not locally defined and with a wall surface for which they are not instantaneously available. Second, the results support the notion that circular heading is specified by the motions of a few elements. In Experiment
2, when movement was known to be parallel to a ground plane, just two dots were sufficient for accurate judgments of circular heading. This is consistent with our analysis that the motions of two elements over two frames specify circular heading. Warren, Blackwell, Kurtz, Hatsopoulos, et al. (1990) examined the effect of number of frames by manipulating the lifetime of individual elements in high-density flow displays of a ground surface. With a two-frame dot life, circular heading judgments were as accurate as in the present study, consistent with the theory. On the other hand, the dependence of heading bias on environmental structure suggests that observers are influenced by other properties of the flow field. One possibility is that unbiased estimates of the center of rotation from vector normals depend on vector lengths, which were affected by the restricted field of view in the present experiments.

In sum, the results demonstrate that observers perceive circular heading in a variety of environments with sufficient accuracy to allow predictive control of locomotion.

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